# Optimal tracking as a framework for normative synthesis of sensory networks

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## ❖ INTRODUCTION

- Sensory networks extract information about external stimuli through their activity.
- The sensory modality of choice is the olfactory system.
- We use a normative, top-down modeling framework motivated by optimal control theory.

## ❖ OBJECTIVES

1. Can we systematically phrase sensory tasks as optimization problems?
2. Can we identify biologically plausible neural architectures and functions from solution of these problems?
3. Are the models predictive of observations made in biological networks?

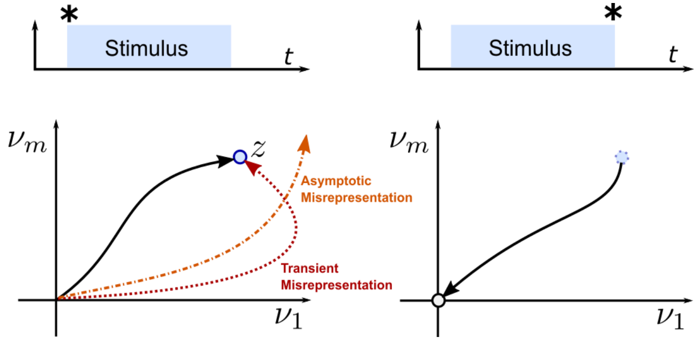
# ❖ MATHEMATICAL FORMULATION & SIMULATION RESULTS

## Premise:

High-dimensional chemical stimuli $\xrightarrow[\text{network}]{\text{Sensory}}$ Low-dimensional latent representation

**Latent space:** $\boldsymbol{\nu}(t) = [\nu_1(t), \dots, \nu_m(t)]$

Stimulus presentation    Stimulus withdrawal




Dynamical decoder maps neural activity to latent representation

## Performance Criteria:

$$\mathbb{J}(\mathbf{x}) = \int_0^T \frac{1}{2}\left[(\boldsymbol{\nu} - z)^T \mathbf{Q}(\boldsymbol{\nu} - z) + \mathbf{x}^T \mathbf{S}\mathbf{x} + \dot{\mathbf{x}}^T \mathbf{R}\dot{\mathbf{x}}\right] dt$$

Error in latent representation | Energy of response | Fluctuations in response

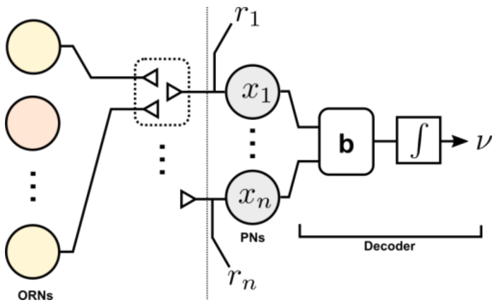
Neural activity that produces accurate latent representation in an energy efficient manner.

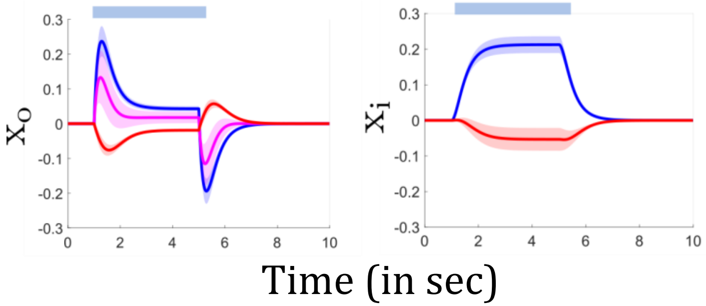
## • PROBLEM 1: SENSORY DETECTION

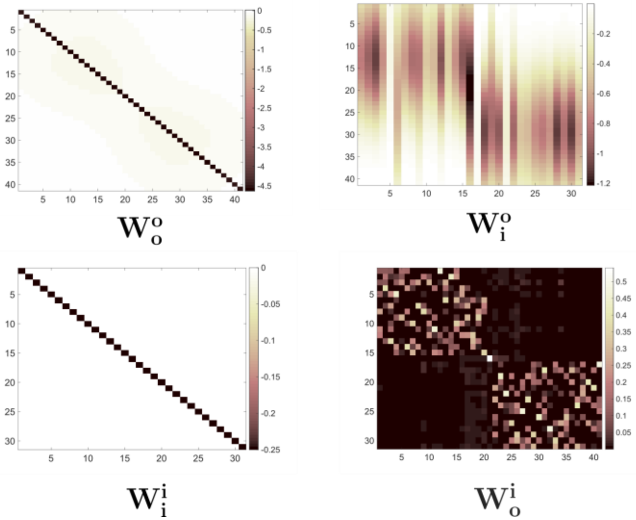
**Optimization framework:**

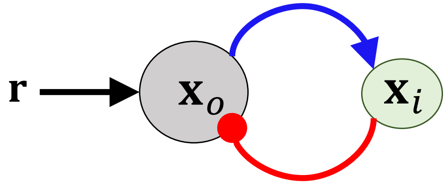
$$\underset{\mathbf{x}}{\operatorname{argmin}}\ \mathbb{J}_1(\mathbf{x}) = \lim_{T\to\infty} \mathbb{J}(\mathbf{x})$$
$$\text{Subject to } \dot{\boldsymbol{\nu}} = -a\boldsymbol{\nu} + \mathbf{b}\mathbf{x}$$

After some numerical optimization…




$$\dot{\mathbf{x}}_o(t) = \mathbf{W}_i^o \mathbf{x}_i(t) + \mathbf{W}_o^o \mathbf{x}_o(t) + \mathbf{r}(t)$$

$$\dot{\mathbf{x}}_i(t) = \mathbf{W}_i^i \mathbf{x}_i(t) + \mathbf{W}_o^i \mathbf{x}_o(t)$$

## ❖ MATHEMATICAL FORMULATION & SIMULATION RESULTS

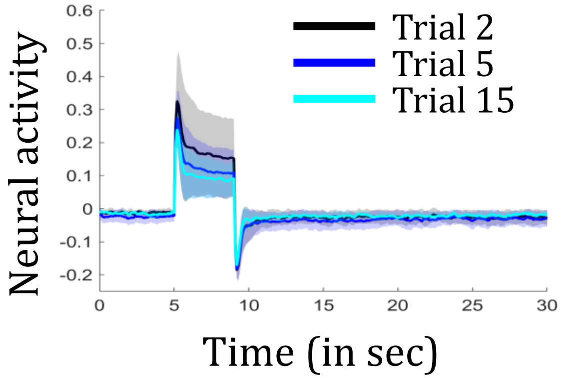
### • PROBLEM 2: NEURAL ADAPTATION

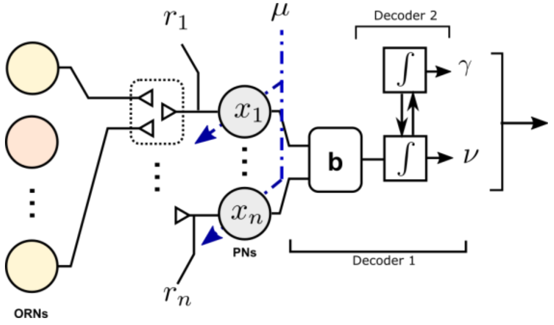
**Optimization framework:**

$$\underset{\mathbf{x}}{\operatorname{argmin}}\ \mathbb{J}_2(\mathbf{x}) = \mathbb{J}(\mathbf{x})$$

$$\text{Subject to}\quad \tau_\nu \dot{\boldsymbol{\nu}} = -a_\nu\,(\boldsymbol{\nu} - \boldsymbol{\gamma}) + \mathbf{bx}$$
$$\tau_\gamma \dot{\boldsymbol{\gamma}} = -a_\gamma \boldsymbol{\gamma} + \beta \boldsymbol{\nu}$$

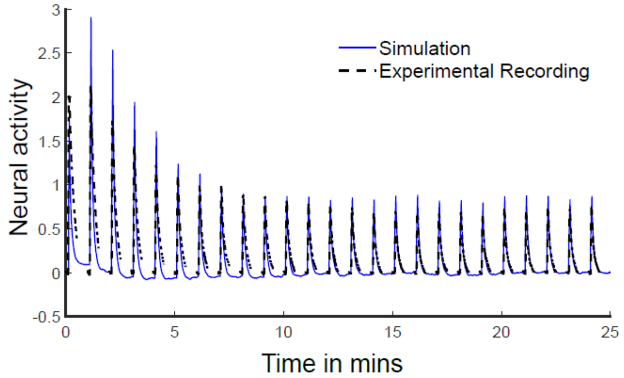
Amplitude of peak response is found to decrease exponentially over repeated trials.




$$\dot{\mathbf{x}}(t) = \boldsymbol{\mu}(t) + \boldsymbol{W_s}\int_0^t e^{-a(t-\tau)}\mathbf{x}(\tau)d\tau$$
$$+\boldsymbol{W_f}\mathbf{x}(t) + \mathbf{r}(t)$$

Post-processing the optimal solution yields response patterns as observed in *C. elegans.*




### • PROBLEM 3: LEARNING IN SENSORY NETWORKS

**Optimization framework:**

$$\underset{\mathbf{x}}{\operatorname{argmin}}\ \mathbb{J}_3(\mathbf{x}) = \mathbb{J}_1(\mathbf{x})|_{T=T_f} + \mathbb{J}_{T_f}(\mathbf{x})$$

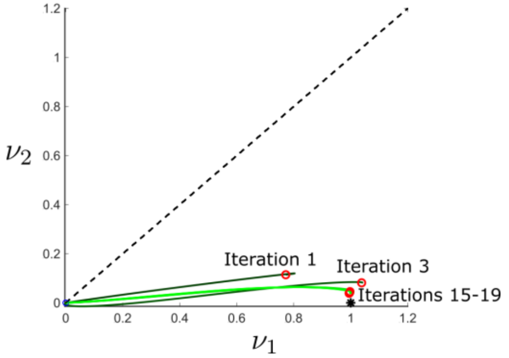
$$\text{Subject to}\quad \dot{\boldsymbol{\nu}} = -a\boldsymbol{\nu} + \mathbf{bf}(\mathbf{x})$$

The nonlinear optimization problem is solved by iterative methods motivated by Pontyagrin's Maximum Principle.

Dynamics of learning over trials:

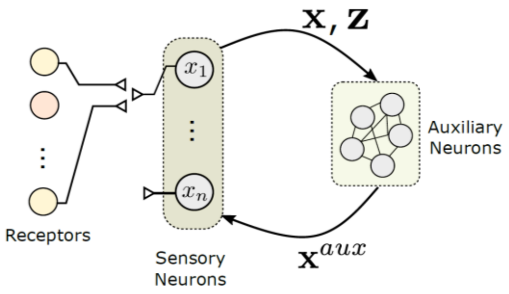
$$\dot{\mathbf{x}}_k(t) \leftarrow (1-\eta_k)\dot{\mathbf{x}}_{k-1}(t) + \eta_k \Delta\dot{\mathbf{x}}_k(t)$$

Convergence of the iterative algorithm indicates that the network has learnt an optimal strategy.



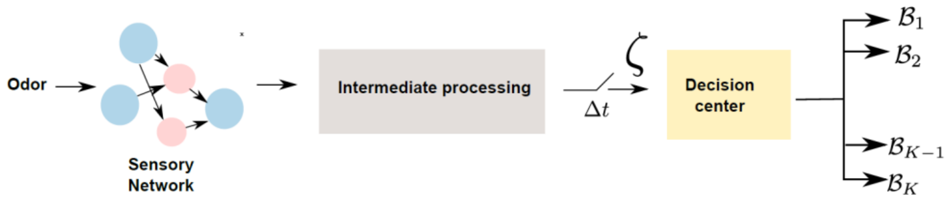

Dynamics of auxiliary 'critic' population:

$$\dot{\mathbf{x}}_k^{aux}(t) = W_k^s(t)\int_0^t e^{-a(t-\tau)}F(\mathbf{x}_k)\mathbf{x}_k^{aux}(\tau)d\tau +$$
$$W_k^f(t)\mathbf{x}_k^{aux}(t) + W_k^\theta(t)h(t,\mathbf{x}_k(t),\mathbf{z})$$



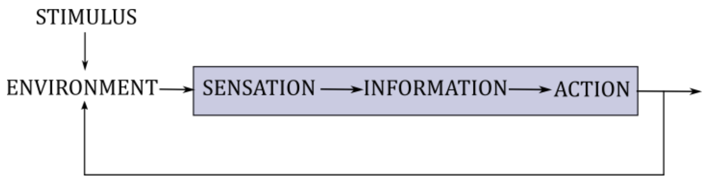
## ❖ DISCUSSIONS

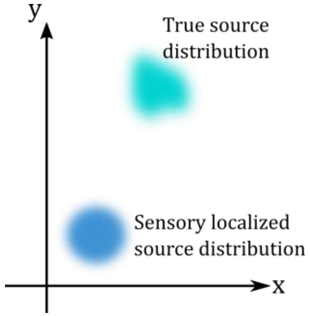
- The model can make predictions for sensory neural coding occurring at population level as well as single neuron level and along different timescales.
- The latent state variables can be used to drive behavioral decision making via a probabilistic decoder.




## ❖ FUTURE SCOPE

- Investigating neural implementation of behavioral decision making.
- Modeling sensorimotor transformations using this framework and analyzing performance in comparison to living organisms.
- On chip implementation of the developed framework.

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